How Horns Work Revisited

This noted audio expert shares with us his experiences and insights into horn operation.

I read with considerable interest the recent series of articles on "How Horns Work" by Bjørn Kolbrek (March and April '08). Having spent the better part of my life studying these devices, I was anxious to read this discussion. I was somewhat disappointed to find so many errors in the discussion, particularly regarding my own work. I had considered responding to these errors individually, but then decided that this may not be the best approach. The discussion could become contentious, which might lead to arguments that were mostly semantic. What I have decided to do instead is to write how I believe horns and waveguides work, and the readers can sort out the errors for themselves.

Historically, horns served a very specific function, and it is important to understand this background because it highlights the motivation for the vast amount of work that I will attempt to shed new light on. That function was to improve the radiation efficiency of the audio playback system. (Please note that there is a distinct difference in the use and function of a musical instrument horn and the same device when used for music reproduction. I do not pretend to know what makes a good musical instrument horn, although I have read most of Arthur Benade's work, finding it, as expected, to be inapplicable to a reproduction horn. So please do not take anything that I say here as a condemnation or critique of the theory of musical instrument horns.)

Early playback systems had a serious problem generating sufficient SPL to be audible in the vast majority of situations. Efficiency was key to an effective design and the horn was well suited to this purpose. However, in a world where you have an almost unlimited availability of power to drive a reproduction device, you might want to question what use efficiency has in our current situation. Not that efficiency is unimportant or a bad thing, only that it is not really a major criterion in a modern design (there are exceptions such as in very large PA systems, but that is not what I am talking about here).

Efficiency is what Webster was concerned with, and nothing more. He wanted to know how much gain a device would have, and his approach works fairly well in that context. His work, however, has been extended well beyond the domain for which it is useful and into areas for which it is not even remotely applicable.

Webster assumed plane wave propagation in his analysis. Most people know this; however, very few ever stop to consider the implications. My favorite physics writer Phillip Morse (who will come up again) did discuss Webster's assumptions in his famous texts *Methods of Theoretical Physics*[1]. In these texts (two volumes) he defines precisely where Webster's assumptions are valid, namely:

$$\left|\frac{d}{dx}\sqrt{S_0 \cdot e^{m \cdot x}}\right| << 1.0$$

where S_0 is the throat area and m is the flare rate. This is actually an extremely limiting restriction as I will show.

Consider **Figs. 1** and **2** (from *Audio Transducers*[2]). The **Fig. 1** plot shows the curve for an exponential horn $y_0 \cdot e^{x\frac{m}{2}}$ as an example, while the **Fig. 2** curve shows a plot of $\frac{d}{dx}\sqrt{S_0 \cdot e^{m \cdot x}}$. The region where Webster's assumptions are valid, according to Morse, lies where x < 15cm, or best case when x < 20cm (<< is usually interpreted to mean < .1). Beyond these lengths (x is the axis along the horns length), you can no longer rely on Webster's equation to be accurate. A line drawn from the center of the throat to its tangent point on the wall contour is shown in **Fig. 1**. This intersection happens at just about the point where the Webster's horn equation loses its validity according to the Morse criteria.

This coincidence highlights an interesting point, namely, that whenever the walls of the horn recede beyond the line where they can be illuminated by the center of the throat (as shown by the straight line), then the Horn equation fails to be reliable. This also happens to be exactly the same point where the wave-fronts would need to diffract if they are to remain in contact with the walls. It is exactly this inability of the horn equation to account for internal diffraction that is at the core of its problems. When there is no internal diffraction, then the horn equation will probably work well, but if there is diffraction, then you must seriously question Webster's approach.

How is it, then, that the horn equation has apparently worked so well in the field for all these years? For "bulk" quantities, such as the input impedance, it does work well. This is because the acoustic impedance is an average of the wave-front details across the waveguide. As long as you are not concerned with these detailed motions, such as in an impedance calculation, then Webster's equation works satisfactorily.

The Horn Equation does an admirable job of predicting the impedance of a wide variety of horn contours, which is completely in line with Webster's goal of finding the "gain" of the device. And it does this through a fairly simple calculation procedure. What's not to like? But the price that you must pay for this simplicity is a complete lack of knowledge of the wave-fronts details across the contour—the horn equation is blind to these details.

DIRECTIVITY

For a variety of reasons, directivity control is a highly desirable feature in loudspeaker systems, and such control is basically unavailable without the use of a waveguide. Much work has been done on directivity control in a waveguide, but much of it is misguided.

You can draw a complex set of wave-fronts on a piece of paper and connect them with a contour, but that does not mean that real sound waves would necessarily follow these drawings. Sound waves have a mind of their own and they *will* go wherever they want, no matter how much hand waving you do. Take, for example, defining horn shapes based on the assumption that the wave-fronts propagate with a constant radius.

This assumption yields some very nice looking curves and horns. Unfortunately, it is physically impossible for a wave-front to propagate in such a manner—it violates every concept in physics from Huygen's Principle to Green's Theorem. The wave-fronts in the real device *will not* follow the wave-fronts as drawn on the paper. It's just not that simple.

Furthermore, a horn contour that is based on a geometrical "plot" of how the wave-fronts will progress relies on faith that the wave-fronts will actually propagate that way. This naïve faith is no way to proceed in a scientific study of a real device. You have no option but to search for a mathematical description that defines precisely *how* the wave-fronts actually will propagate using physics, not naïvely prescribing a situation that you *hope* will occur.

To predict or design for directivity, you must know exactly what the wave-fronts look like at the mouth of the waveguide, and this is not so easy—certainly not as easy as just calculating the impedance. This is precisely the point where I found myself in the 80s. I wanted to do accurate directivity control in a horn, but I realized that none of the mathematical descriptions available at that time could achieve this.

I had studied the early work of Morse in *Vibration and Sound*[3] and found his concept of 1P (for one parameter) waves compelling. It was natural that I looked for solutions along these lines. Making a long story short, to be 1P means that the waves must be described by a coordinate system of the so-called orthogonal variety.

Orthogonal coordinates are well known, and Morse's *Methods. . .* [1] devotes an entire chapter to this topic (chapter V, my favorite, one of the most elegant discussions in all of my physics literature). It can be shown that you can define exactly 13 (and only 13) coordinate systems that are orthogonal in three dimensions. However, of these 13, only 11 allow solutions to the wave equation. In other words, sound waves can be described mathematically by only 11 coordinate systems.

It became readily apparent to me that only in these 11 coordinate systems could you ever hope to find a mathematical description along the lines of the 1P idea. The problem is that of these 11 only two actually allow for true 1P behavior. The others, while still orthogonal, are dimensionally coupled spectrally through their so-called separation constants and end up not being 1P. In essence, this means that the wavefronts "leak" energy from one coordinate into the others—basically the diffraction aspect that I mentioned earlier.

THE COUP DE GRÂCE FOR 1P

It is interesting to note that the two (reasonable) 1P possibilities are the radial coordinate in spherical coordinates and the radial coordinate in cylindrical coordinates. The two horns that are defined by these coordinates turn out to be exactly the same two horns that Putland discusses in his paper on Webster's equation[4]—which deserves a short diversion. "Didn't Putland show that any solution of Webster's equation is a 1P solution?" Yes, that's absolutely correct, and he also proved that Webster's equation is exactly correct for only two horns, not coincidentally the same two that I defined above through a completely different line of reasoning.

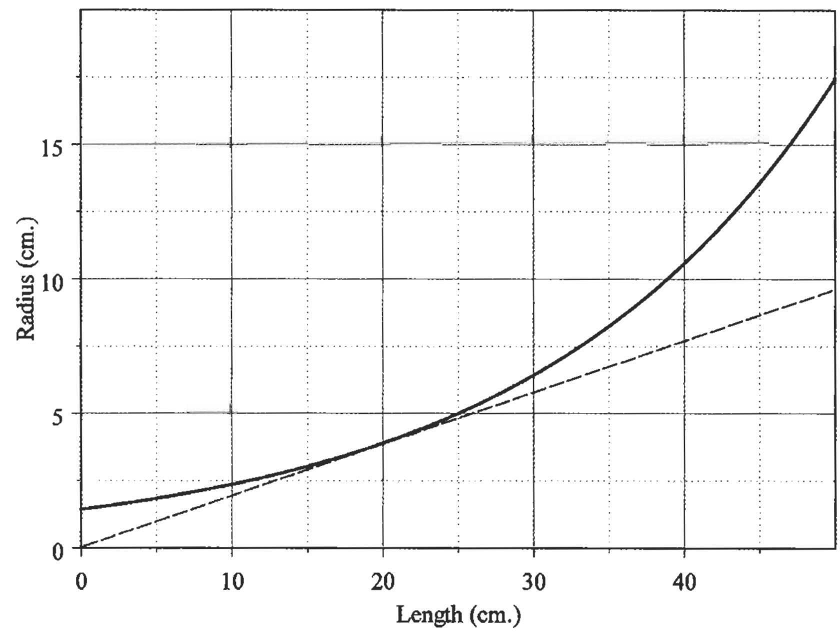

Figure 1: Typical exponential horn contour.

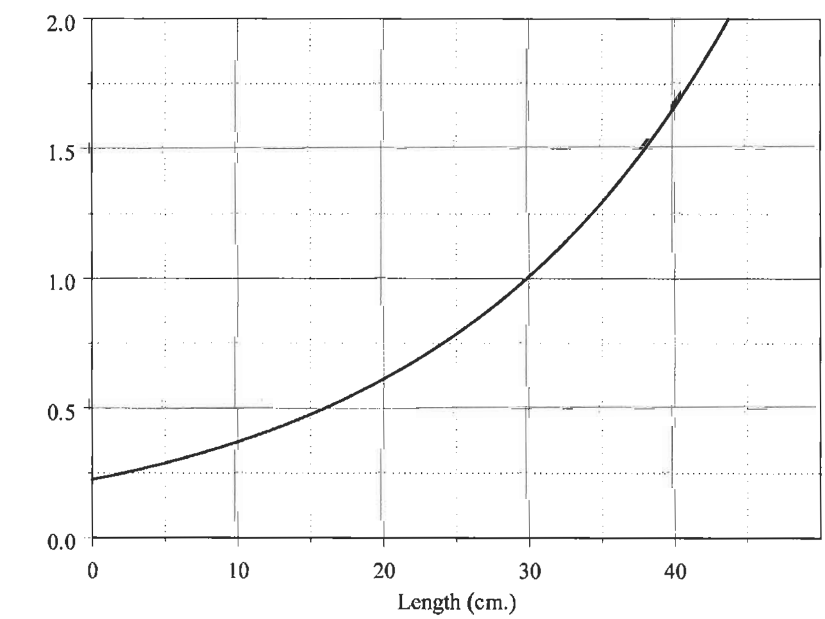

Figure 2: Plot of equation.

I had actually arrived at this exact same conclusion some years before Putland (in *Acoustic Waveguide Theory*[5]), but I guess he must have missed it because he didn't mention it. The claim made by Mr. Kolbrek that "Putland later showed that this was not the case," is incorrect because he did no such thing. Putland also did not indicate a recognition of the existence of the higher order solutions (they aren't predicted by Webster's solutions, but do exist in the wave equation approach—it is these details that make all the difference!).

You see, in order for a 1P wave to propagate within a 1P device, it must be excited by a purely 1P source wave-front. Any other wave-front will excite higher order modes (HOMs), and the wave propagation will no longer be 1P. In the case of a conical horn (from spherical coordinates), that's a spherical cap that vibrates radially (normal to the curved surface). Unfortunately, such a source device does not actually exist; you could hypothesize making one, but they don't actually exist today.

A dome loudspeaker vibrates axially, not radially, and so it is not 1P. A conical horn on a dome source is not 1P. For the horn in cylindrical coordinates the source must be a radially vibrating cylindrical section, which, once again, does not exist in practice.

So while Putland may have been correct in his *JAES* article[6], this fact is of no practical use because 1P devices can only exist in a hypothetical world, and real-world devices are never 1P. It is no coincidence that both Morse and I ceased to use the concept of 1P after studying the problem because we both realized that such an ideal solution was only of academic interest in a hypothetical situation.

What I have said thus far:

• Unless you are only interested in the impedance of a device, Webster's horn equation is not of much use, and even then it works correctly only in a hypothetical situation of no practical interest. To me, any proposition that uses the horn equation at its foundation needs to be viewed with extreme skepticism.

IMPEDANCE (AGAIN)

"Well, at least the horn equation is good for predicting the shapes that one needs in order to achieve good impedance loading—at least at low frequencies!" Sure, except that it turns out that virtually any shape connecting a given throat and mouth area will yield approximately the same impedance—within a dB or so. In the long wavelength region where the horn equation is valid, shape is simply not a significant factor; the waves don't see shape. Basically, they only see the inlet and the outlet areas and the distance between them and everything else is of secondary importance. But, if something secondary is important, then it is almost certain that Webster's equation won't apply.

THE EXACT SOLUTION OF THE WAVEGUIDE EQUATIONS

Contrary to some widely propagated errors (one of which is in Mr. Kolbrek's article), I actually did develop an exact solution for a waveguide using the Oblate Spheroidal (OS) coordinate system[5, 6]. It was in these AES papers that I came to realize the importance of HOMs, which the horn equation fails to account for and which make true 1P behavior unrealistic. All horns and waveguides in the real world have HOMs, which produce the complex wave motion that prevents us from prescribing it. Whenever we attempt to force the wave-front to travel a path that it doesn't want to, it gets back at us by generating HOMs.

The advantage of waveguides based on the orthogonal coordinate geometries is that it can be proven that they will generate the least HOMs of any contour connecting the same input and output apertures. No shape can do better. That, to me, is the attraction to the approach and the OS waveguide for a standard circular source.

It is interesting to note that if you draw the OS contour as in **Fig. 1**, that a line from the origin to the walls is never tangent to the walls as it is in the exponential (and virtually every other) horn.

I have solved the waveguide equations in just about all of the geometries for which the theory applies and designed and built waveguides in them. They all work well, but with different constraints on the polar patterns, source configurations, and so on. Different coordinate systems can also be combined such as in my *Bi-Spheroidal*™ waveguide (patent #7,068,805) in order to achieve specific objectives.

Readers interested in more detail on waveguide theory can find them in "Chapter 6—Waveguides" of my book *Audio Transducers*[2](available through Old Colony).

I would like to point out why I coined the term "waveguide." It was done to highlight the fact that the horn equation was not used and as such a new name was appropriate. Unfortunately, usage of this term has gotten out of control and it is now applied to whatever anyone wants to apply it to.

With a waveguide, a complete knowledge of the wave-front is available at any point, the most useful surfaces being the mouth and the throat apertures—the interfaces. If you have a desired directivity pattern, then the wave-front shape required in the mouth aperture can be calculated—assuming that such a pattern is obtainable within the given mouth size. The calculations will, however, define what the mouth size needs to be in the event that it is too small or what must be given up for a fixed mouth size. With this knowledge of the mouth wave-front, you can then calculate the wave-front required at the throat. Now, in principle, you could fabricate a phase plug that achieves the required throat wave-front (it's not likely to be a plane wave). Because the wave shape may need to change with frequency in order to achieve the desired polar pattern, there is no guarantee that such a solution is possible or practical. But it is guaranteed that no other design could do any better because with waveguide theory you can get right up to the limits of what the physics of the situation will allow. Such a detailed design is simply not possible with horn theory.

DIFFRACTION

Diffraction—that aspect of horns that horn theory cannot deal with—turns out to be of predominate importance to sound quality. Diffraction in a waveguide is a bad thing, a very bad thing as it turns out. It is not readily masked in the ear, and any masking that does exist decreases with sound level[7], meaning that diffraction becomes more audible at higher sound levels—the exact opposite of nonlinear distortion. Diffraction is what you hear in a waveguide as harshness or poor sound quality. Reduce the diffraction and the device simply sounds better.

Diffraction can occur in a number of places including the phase plug. When the waveguide is not presented with the proper wave-front as prescribed by its specific geometry, then a form of diffraction occurs in that HOMs are generated. Any form of discontinuity anywhere in the device can produce this problem.

In my experience no amount of attention to the detail of the throat geometry and its wave-front is unwarranted. Even small discontinuities in the throat matching or errors in the phase plug design will generate diffraction in the form of HOMs, which then become amplified by the waveguide. Specific attention to the phase plug is of paramount importance (patent # 7,095,868 and patents pending).

THE PIÈCE DE RÉSISTANCE

Only quite recently I had an epiphany regarding waveguides versus horns. Horn theory is concerned only with the rate of change of area as it relates to the impedance presented by the device. No consideration is actually given to the shape of the boundary beyond the effect that it has on the area. Waveguide theory on the other hand is the exact opposite in that no explicit consideration is given to the area—it is what it is—and only the shape of the boundary matters.

The key point is that all of the 11 orthogonal coordinate systems (those that allow for expanding waveguides as opposed to parabolic ones) have one very important characteristic in common: every one of them has exactly the same shape boundary, either straight or of the form

$$y(x)\sqrt{y_0^2 + \left(x\,\mathrm{Tan}(\theta_0)\right)^2}$$

where y_0 is the throat radius and θ_0 the wall angle. This curve is known as a catenoid, the minimum length of a curve joining two points with prescribed angles at each end point, with the one at y_0 being zero (modifications for non-zero entry angles are possible). A straight line is the catenoid when the two points have the same angles.

In short, horn theory is concerned with the rate of change of area and impedance while waveguide theory is concerned with the rate of change of the boundary and the ensuing wave-shape. Waveguide theory seeks to minimize the generation of diffraction within the device but Horn theory, unfortunately, does not even consider it.

CONTROLLING HOMs

Having realized the importance of the HOMs, you can do several things that are quite separate from the waveguide design to minimize these undesirable characteristics. I have learned that the inclusion of open cell foam within the body of the waveguide will reduce the HOMs much more than the primary wave, thus yielding a pronounced net benefit in sound quality (patent pending).

MY POV

After 30 years of study, I have come to several conclusions that I will highlight here:

1. The horn equation is of no practical use in any useful device.
2. Impedance calculations are of marginal utility—I never do them for my designs—and the concepts of loading, cutoff, and so on are all obsolete holdovers from horn theory.
3. Nonlinear distortion in a waveguide is irrelevant. This comes from the fact that this distortion is of very low order and as such is inaudible[8,9].
4. Waveguides are primarily useful for directivity control and are the only practical way to achieve this critical design objective.
5. Sound quality goes in inverse relation to the presence of HOMs, and attention to detail in this regard is essential.
6. Waveguides are not very useful for low-frequency transducers. The directivity control function of the waveguide is defeated when its size is small compared to the wavelength; efficiency and low distortion are not that important at these frequencies; and I can get better efficiency and lower distortion using a more efficient approach such as an acoustic lever (patent # 6,782,112) or a bandpass design[10]. There are some situations where a horn's good efficiency over a larger bandwidth than levers or bandpass designs can be useful, but there are also serious trade-offs both ways. I have not found horns to be cost and space efficient at low frequencies in any of my work.

There will be a multitude of people who will disagree with these conclusions and they will use all kinds of arguments about what "sounds better." That's fine, they have every right to their subjective opinions. But mathematics is not subject to opinion nor open to debate, except with alternative *mathematical* arguments. Science is not a democracy that can be voted on with the popular opinion becoming reality (except for evolution).

The reader will note many areas where I disagree with Mr. Kolbrek, but mostly I disagree with continuing to propagate concepts that are not based on reliable science, having been derived from a misguided application of horn theory beyond its very limited applicability.

I would like to thank Mr. Kolbrek for his good historical discussion and many good references, some of which were previously unknown to me. I hope that he will find my arguments compelling enough to embrace waveguide theory as the only viable approach to the problems at hand. *aX*

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Bjørn Kolbrek responds:

I would like to thank Dr. Geddes for his interest in my article, and for his comments. He raises many valid points. I will attempt to address the key ones.

I should perhaps begin by reiterating the intent of my article, which is clearly outlined in the title ("Horn Theory: An Introduction") and the opening paragraph: The article ". . . reviews the basic assumptions behind classical horn theory as it stands, presents the different types of horns, and discusses their properties." It is therefore unreasonable to expect it to give a complete account of acoustical waveguide theory.

That the Webster horn equation is nothing more than an approximation was pointed out in my article, and as an approximation it can only deal with certain aspects of what it describes. Many of the limitations of the horn equation have been known since the 1920s, as a reading of the references I have quoted will show. As Dr. Geddes says, Webster's horn equation deals with acoustical impedance. I have never claimed otherwise. I do not, however, believe that the application of the horn equation is as limited as Dr. Geddes says. It is true that it is only completely valid within a small range. But the limits of approximate validity can be stretched quite a bit, and several investigators have shown that the throat impedance of a horn can be calculated with good accuracy (compared to measured results), if you abandon the plane wave assumption and make use of a curved isophase wave-front area expansion in the calculations.

However, if you want to calculate directivity and higher order effects, then I agree that the horn equation is not applicable. I believe I made that point clear in my article. I also tried to show that horn design is not as simple as deriving the profile from an assumed set of wave-fronts. Sound waves will do "whatever they want," and I believe Figs. 15 and 16 from part one of my article show this clearly enough.

Dr. Geddes has rejected the horn equation as having no practical use. From that point of view, every treatment of horn theory based on this equation will by definition be looked upon as being "full of errors." I believe that many of the errors he has found in my article come from the fact that I did not reject the horn equation completely.

That I have made factual errors is clear. I have obviously misunderstood the Geddes/Putland debate and some of Dr. Geddes' work. It was perhaps also a mistake to include the OS waveguide in a discussion on classical horn theory, because this device is best analyzed using waveguide theory. For these errors I offer Dr. Geddes my sincere apologies.

That said, I don't understand how Dr. Geddes can claim that all horns that have the same end areas and length will have virtually the same loading characteristics regardless of profile, without giving the conditions under which the assertion holds. It is probably applicable to short horns, in the range where the horn equation is valid. If it is believed to be true for a larger range of horn profiles and sizes, then I would welcome seeing the practical data, because all the published measurements that I am aware of show the opposite. It is true that all horns approach the asymptotic throat acoustical impedance of

$$Z_A = \frac{\rho_0 c}{S_t}$$

but the frequency at which this occurs is not the same for all horn profiles, no matter how much "hand-waving" is done.

When throat impedance (and resistive loading of the driver), distortion, and efficiency are all rejected as unimportant, then classical horn theory does not have much to offer. Designers concerned only about directivity control and higher order effects are better off with waveguide theory. It is, however, my view that classical horn theory still has its uses, but you must keep in mind that you are working with approximations.