Vented-Box Loudspeaker Systems
Part IV: Appendices

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The appendices present a method of calculating the system parameters required to obtain a desired alignment defined by transfer-function polynomial coefficients in the presence of enclosure losses together with diaphragm displacement data for that alignment, a derivation of the parameter-impedance relationships that permit parameter evaluation from voice-coil impedance measurements, and a method of evaluating the amounts of absorption, leakage, and vent losses present in a vented-box loudspeaker system.

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APPENDIX 1
FOURTH-ORDER FILTER FUNCTIONS AND VENTED-BOX SYSTEM ALIGNMENT

General Expressions

The general form of a prototype low-pass fourth-order filter function $G_L(s)$ normalized to unity in the passband is

$$G_L(s) = \frac{1}{1 + a_1 sT_0 + a_2 s^2T_0^2 + a_3 s^3T_0^3 + s^4T_0^4} \quad (55)$$

where T_0 is the nominal filter time constant and the coefficients a_1, a_2, and a_3 determine the actual filter characteristic.

Tables of filter functions normally give only the details of a low-pass prototype function; the high-pass and bandpass equivalents are obtained by suitable transformation. For the high-pass filter function $G_H(s)$, the transformation (retaining the same nominal time constant) is

$$G_H(sT_0) = G_L(1/sT_0). \quad (56)$$

This leads to the general high-pass form of Eq. (20):

$$G_H(s) = \frac{s^4T_0^4}{s^4T_0^4 + a_1 s^3T_0^3 + a_2 s^2T_0^2 + a_3 sT_0 + 1}. \quad (57)$$

Study of the magnitude-versus-frequency behavior of filter functions is facilitated by the use of the magnitude-squared form

$$|G_H(j\omega)|^2 = \frac{\omega^8T_0^8}{\omega^8T_0^8 + A_1\omega^6T_0^6 + A_2\omega^4T_0^4 + A_3\omega^2T_0^2 + 1} \quad (58)$$

where

$$A_1 = a_1^2 - 2a_2$$
$$A_2 = a_2^2 + 2 - 2a_1a_3$$
$$A_3 = a_3^2 - 2a_2. \quad (59)$$

Using Eq. (58) it can be shown that the magnitude response of G_H is down 3 dB, i.e., $|G_H|^2 = 1/2$, at a frequency f_3 given by

$$f_3/f_0 = d^{1/2} \quad (60)$$

where

$$f_0 = 1/(2\pi T_0) \quad (61)$$

and d is the largest positive real root of the equation

$$d^4 - A_1d^3 - A_2d^2 - A_3d - 1 = 0. \quad (62)$$

Coefficients of Some Useful Responses

Butterworth Maximally Flat Amplitude Response (B4)

This well-known response is characterized by [10], [18]

$$a_1 = (4+2\sqrt{2})^{1/2} = 2.6131$$
$$a_2 = 2+\sqrt{2} = 3.1412$$
$$a_3 = a_1 = 2.6131$$
$$A_1 = A_2 = A_3 = 0$$
$$f_3/f_0 = 1.0000$$

Bessel Maximally Flat Delay Response (BL4)

The normalized roots are given in [19]. They yield

$$a_1 = 3.20108 \quad A_1 = 1.4638$$
$$a_2 = 4.39155 \quad A_2 = 1.2857$$
$$a_3 = 3.12394 \quad A_3 = 0.9759.$$
$$f_3/f_0 = 1.5143$$

Chebyshev Equal-Ripple (C4) and "Sub-Chebyshev" (SC4) Responses

These responses are both described in [14]; the $C4$ responses are further described in [32]. The pole locations may be derived from those of the Butterworth response by multiplying the real part of the Butterworth pole by a factor k which is less than unity for the $C4$ responses and greater than unity for the $SC4$ responses. The filter-function coefficients are then given by

$$a_3 = \frac{k(4+2\sqrt{2})^{1/2}}{D^{1/4}}$$
$$a_2 = \frac{1+k^2(1+\sqrt{2})}{D^{1/2}}$$
$$a_1 = \frac{a_3}{D^{1/2}}\left[1 - \frac{1-k^2}{2\sqrt{2}}\right] \quad (63)$$

where

$$D = \frac{k^4+6k^2+1}{8}.$$

For the $C4$ responses, the passband ripple is given by

$$\text{dB ripple} = 10\log_{10}[1+K^4/(64+28K+80K^2+16K^3)] \quad (64)$$

where

$$K = 1/k^2 - 1.$$

Quasi-Third-Order Butterworth Responses (QB3)

This class of response is described in [10] and [32]. In this paper, the response is varied as a function of the parameter B given by

$$B = A_3^{1/2}. \quad (65)$$

The other coefficients are given by

$$A_1 = A_2 = 0$$
$$a_2 > 2+\sqrt{2}$$
$$a_1 = (2a_2)^{1/2}$$
$$a_3 = (a_2^2+2)/(2a_1). \quad (66)$$

Because the direct relationships between B and the a coefficients are very involved, the range of responses is computed by taking successive values of a_2 and then computing a_1, a_3, A_3, and B.

Other Possible Responses

Other fourth-order responses which can be obtained with the vented-box system include transitional Butterworth–Thompson [18], transitional Butterworth–Chebyshev [30], Thiele interorder [31], and degenerated Chebyshev [11].

The degenerated Chebyshev responses of the second kind ($DT2$) described by Nomura [11] look particularly appealing for cases where a smooth bass lift (similar to an underdamped second-order response, but with a steeper cutoff slope) is desired. Nomura's design parameters are readily convertible into those of this paper.

Computation of Basic Alignment Data

The basic alignment data are obtained by using the coefficient–parameter relationships given by Eqs. (21)–(24). The steps are as follows.

1) For a given response and value of Q_L calculate

$$c_1 = a_1 Q_L$$
$$c_2 = a_3 Q_L. \quad (67)$$

2) Find the positive real root r of

$$r^4 - c_1 r^3 + c_2 r - 1 = 0. \quad (68)$$

3) Then, using Eqs. 60–62 to obtain f_3/f_0, the alignment parameters are

$$h = r^2$$
$$f_3/f_S = h^{1/2}(f_3/f_0)$$
$$\alpha = a_2 h - h^2 - 1 - (1/Q_L^2)(a_3 h^{1/2} Q_L - 1)$$
$$Q_T = hQ_L/(a_3 h^{1/2} Q_L - 1). \quad (69)$$

For infinite Q_L the above expressions reduce to Thiele's formulas:

$$h = a_3/a_1$$
$$f_3/f_S = h^{1/2}(f_3/f_0)$$
$$\alpha = a_2 h - h^2 - 1$$
$$Q_T = 1/(a_1 a_3)^{1/2}. \quad (70)$$

Computation of Displacement Maxima

Eq. (14) may be written in the generalized form

$$X(s) = \frac{b_1 s^2 T_0^2 + b_2 s T_0 + 1}{s^4 T_0^4 + a_1 s^3 T_0^3 + a_2 s^2 T_0^2 + a_3 s T_0 + 1} \quad (71)$$

where T_0, a_1, a_2, and a_3 are given by Eqs. (21)–(24) or by the alignment specification and

$$b_1 = 1/h$$
$$b_2 = 1/(h^{1/2} Q_L). \quad (72)$$

The magnitude-squared form of this expression is

$$|X(j\omega)|^2 = \frac{B_1\omega^4 T_0^4 + B_2\omega^2 T_0^2 + 1}{\omega^8 T_0^8 + A_1\omega^6 T_0^6 + A_2\omega^4 T_0^4 + A_3\omega^2 T_0^2 + 1} \quad (73)$$

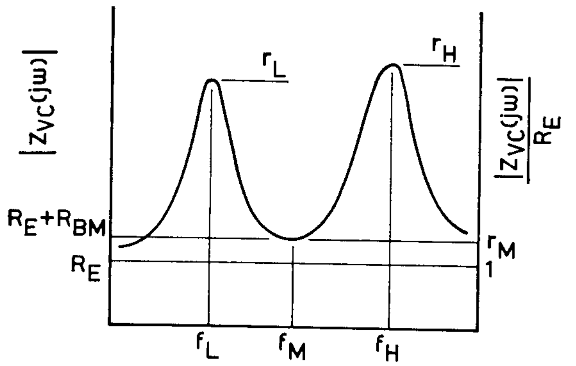

Fig. 20. Voice-coil impedance magnitude of vented-box loudspeaker system as a function of frequency.

where the A_i coefficients are given by Eq. (59) and

$$B_1 = b_1^2$$
$$B_2 = b_2^2 - 2b_1. \qquad (74)$$

The value of $|X(j\omega)|_{\max}^2$ for any alignment is found by differentiating Eq. (73), setting the result equal to zero, solving for the value of $\omega^2 T_0^2$, and then replacing this solution in Eq. (73) and evaluating the expression. There are always at least three frequencies of zero slope for Eq. (73): zero, near f_B, and above f_B. For the extreme $C4$ alignments, there is a fourth frequency, below f_B. The first of these frequencies gives unity displacement; the second is not of interest because it gives a displacement minimum. The third frequency gives the displacement needed to evaluate the displacement-limited power capacity for bandwidth-limited drive conditions. The procedure is as follows.

1) For a given alignment and value of Q_L, calculate

$$C_4 = (1/2B_1)(A_1B_1 + 3B_2)$$
$$C_3 = (1/B_1)(A_1B_2 + 2)$$
$$C_2 = (1/2B_1)(3A_1 + A_2B_2 - A_3B_1)$$
$$C_1 = (1/B_1)(A_2 - B_1)$$
$$C_0 = (1/2B_1)(A_3 - B_2). \qquad (75)$$

2) Find the largest positive real root G of

$$G^5 + C_4G^4 + C_3G^3 + C_2G^2 + C_1G + C_0 = 0. \qquad (76)$$

(The normalized frequency of maximum passband displacement is then $f_{X\max}/f_0 = G^{1/2}$).

3) Calculate

$$|X(j\omega)|_{\max}^2 = \frac{B_1G^2 + B_2G + 1}{G^4 + A_1G^3 + A_2G^2 + A_3G + 1}. \qquad (77)$$

The same procedure is used to determine the frequency of maximum displacement below f_B for the extreme $C4$ alignments by finding the smallest nonzero positive real root in 2). The corresponding maximum value of the displacement function magnitude is then determined as in 3).

APPENDIX 2
PARAMETER-IMPEDANCE RELATIONSHIPS

Determination of f_{SB} and α

For infinite Q_L, the steady-state form of Eq. (16) becomes

$$Z_{VC}(j\omega) = R_E + R_{ES}\frac{j(\omega T_S/Q_{MS})(1-\omega^2T_B^2)}{\omega^4T_B^2T_S^2 + 1 + \omega^2[(\alpha+1)T_B^2 + T_S^2] + j(\omega T_S/Q_{MS})(1-\omega^2T_B^2)}. \qquad (78)$$

This expression has minimum magnitude and zero phase when the numerator of the second term is zero, i.e., when $\omega = 1/T_B$. Thus for this case, the frequency f_M of Fig. 20 is equal to f_B. The expression also has zero phase, with maximum magnitude, when the real part of the denominator of the second term is zero, i.e., for

$$\omega^2 = \frac{T_S^2 + (\alpha+1)T_B^2 \pm \sqrt{T_S^4 + (\alpha+1)^2T_B^4 + (2\alpha-2)T_B^2T_S^2}}{2T_B^2T_S^2}. \qquad (79)$$

Let the solution using the plus sign be ω_H^2 and the solution using the minus sign be ω_L^2. Then

$$\omega_H^2 + \omega_L^2 = \omega_B^2 + (\alpha+1)\omega_S^2 \qquad (80)$$

and

$$(\omega_H^2 - \omega_L^2)^2 = \omega_B^4 + (\alpha+1)^2\omega_S^4 + (2\alpha-2)\omega_B^2\omega_S^2. \qquad (81)$$

Combining Eqs. (80) and (81), it can be shown that

$$(\omega_H^2 - \omega_L^2)^2 = (\omega_H^2 + \omega_L^2)^2 - 4\omega_B^2\omega_S^2 \qquad (82)$$

which simplifies to

$$\omega_H^2\omega_L^2 = \omega_S^2\omega_B^2$$

or [10, eq. (105)]

$$f_S = \frac{f_Hf_L}{f_B} \qquad (83)$$

where $f_S = f_{SB}$ is the resonance frequency of the driver for the particular air-load mass presented by the enclosure.

With f_S known, α can be found by rearranging Eq. (80) into

$$\alpha = \frac{f_H^2 + f_L^2 - f_B^2}{f_S^2} - 1. \qquad (84)$$

Alternatively, substituting Eq. (83) into Eq. (80), it is easily shown that [10, eq. (106)]

$$\alpha = \frac{(f_H^2 - f_B^2)(f_B^2 - f_L^2)}{f_H^2f_L^2}. \qquad (85)$$

This expression factors into

$$\alpha = \frac{(f_H + f_B)(f_H - f_B)(f_B + f_L)(f_B - f_L)}{f_H^2f_L^2}. \qquad (45)$$

Approximate Determination of Q_B

From Fig. 3, Z_{VC} will be resistive when the portion of the circuit to the right of R_{ES} is resistive. The steady-state impedance of this portion of the circuit is

$$Z(j\omega) = R_{EL}\frac{(\alpha T_BQ_L)[-\omega^2T_B/Q_L + j\omega(1-\omega^2T_B^2)]}{\omega^4T_B^2T_S^2 + 1 - \omega^2[(\alpha+1)T_B^2 + T_S^2] + j\omega(T_B/Q_L)(1-\omega^2T_S^2)} \qquad (86)$$

At a frequency of zero phase, the magnitude of $Z(j\omega)$ may be evaluated by taking the ratio of either the real or the imaginary parts of the numerator and denominator, because these ratios must be equal. That is, for zero phase,

$$|Z(j\omega)| = R_{EL}(aT_BQ_L)\frac{-\omega^2T_B/Q_L}{\omega^4T_B^2T_S^2+1-\omega^2[(a+1)T_B^2+T_S^2]}$$

$$= R_{EL}(aT_BQ_L)\frac{1-\omega^2T_B^2}{(T_B/Q_L)(1-\omega^2T_S^2)}. \quad (87)$$

Setting the real and imaginary ratios equal in the normal way leads to a very complex set of solutions for the exact frequencies of zero phase. However, it can be seen that the first ratio varies relatively slowly with frequency near ω_B (as indeed does $|Z_{VC}(j\omega)|$) and hence can be expected to have about the same magnitude at the frequency of zero phase ω_M very near to ω_B as it has at ω_B. This gives

$$|Z(j\omega_M)| \approx |Z(j\omega_B)| = R_{EL}. \quad (88)$$

The resistive voice-coil impedance measured at f_M, defined as $R_E + R_{BM}$ in Fig. 20, is thus made up of R_E plus the parallel combination of R_{ES} and R_{EL}. Evaluating this resistance and using Eqs. (5), (7), (8), (10), and (11), it can be shown that

$$Q_L = \frac{h}{a}\left[\frac{1}{Q_{ES}(r_M-1)} - \frac{1}{Q_{MS}}\right] \quad (49)$$

where r_M is $(R_E + R_{BM})/R_E$ as defined in Eq. (48) and Fig. 20. In many cases the $1/Q_{MS}$ term can safely be neglected.

Now, if the two ratios in Eq. (87) are equal at ω_M, the second must give the same value as the first. This requires that

$$\omega_M^2 = \frac{1-aQ_L^2}{T_S^2 - aT_B^2Q_L^2} \quad (89)$$

which may be rearranged to give Eq. (50). The approximation made earlier in Eq. (88) seems justified by Eq. (50) for Q_B values as low as 5, because the difference between f_M and f_B is then at most a few percent. For lower values of Q_B (which are unusual), substantial inaccuracy must be expected. Inaccuracy can also be contributed by a significant voice-coil inductance (see [32]).

APPENDIX 3
MEASUREMENT OF ENCLOSURE LOSSES

Measurement Principle

In this method of measurement the system driver is used as a coupling transducer between the enclosure impedances and the electrical measuring equipment. The driver losses are subtracted from the total measured losses to obtain the enclosure losses. Greatest accuracy is therefore obtained where the driver mechanical losses are small and stable.

The method assumes that R_E remains constant with frequency (i.e., voice-coil inductance losses are negligible), that the individual enclosure circuit losses correspond to Q values of about 5 or more (so that $Q^2 >> 1$), and that any variation with frequency of the actual losses present can still be represented effectively by a combination of the three fixed resistances R_{AB}, R_{AL}, and R_{AP} of Fig. 1.

System Loss Data

From the system impedance curve, Fig. 20, find the three frequencies f_L, f_M, and f_H, and the ratio of the corresponding maximum or minimum impedance to R_E, designated r_L, r_M, and r_H.

Using the methods of Section 7 (Part II) or [32], determine the system compliance ratio a. Measure independently the driver resonance frequency f_S and the corresponding value of Q_{ES} as described in [12] or [32]. The driver mounting conditions for the latter measurements do not matter, because the product f_SQ_{ES} which will be used is independent of the air-load mass present.

Driver Loss Data

Let the symbol ρ be used to define the ratio

$$\rho = (R_{ES}+R_E)/R_E. \quad (90)$$

Because R_{ES} is in fact a function of frequency for real drivers, so too is ρ. Typically the variation is of the order of 2 to 4 dB per octave increase with increasing frequency.

At the resonance frequency of the driver, ρ is the ratio of the maximum voice-coil impedance to R_E which is defined as r_0 in [12]. The value of ρ for frequencies down to f_L may be measured by weighting (mass loading) the driver diaphragm and measuring the maximum voice-coil impedance at resonance for a number of progressively lower frequencies as more and more mass is added. A convenient nondestructive method of weighting is to stick modeling clay or plasticene to the diaphragm near the voice coil.

Unfortunately, there is no comparable simple way to reduce mass or add stiffness which will raise the driver resonance frequency without affecting losses. For simplicity, it is necessary to extrapolate the low-frequency data upward to f_H. This is risky if f_H is more than an octave above f_S but gives quite reasonable results for many drivers.

Under laboratory conditions, it is possible to fabricate a low-mass driver which is "normally" operated with a fixed value of added mass. This mass is selected so that the unloaded driver resonance occurs at a frequency equal to or greater than the value of f_H for the loaded driver in a particular enclosure. In this case the value of ρ can be accurately determined for the entire required frequency range by adding and removing mass.

Measure and plot (extrapolating if necessary) the value of ρ over the frequency range f_L to f_H. Find the values at f_L, f_M, and f_H and designate these ρ_L, ρ_M, and ρ_H.

These measurements should be carried out at the same time and under the same conditions as those for the system loss data above. The signal level should be the same and should be within small-signal limits at all times.

Enclosure Loss Calculation

Define:

$$H = f_H/f_M$$
$$L = f_M/f_L$$
$$F = f_M/(af_SQ_{ES}). \quad (91)$$

Calculate:

$$k_L = \frac{1}{r_L - 1} - \frac{1}{\rho_L - 1}$$

$$k_M = \frac{1}{r_M - 1} - \frac{1}{\rho_M - 1}$$

$$k_H = \frac{1}{r_H - 1} - \frac{1}{\rho_H - 1} \quad (92)$$

$$C_L = Fk_L(L^2 - 1)\left(1 - \frac{1}{L^2}\right)$$

$$C_M = (Fk_M)^{-1}$$

$$C_H = Fk_H(H^2 - 1)\left(1 - \frac{1}{H^2}\right) \quad (93)$$

$$\Delta = \left(H^2L^2 - \frac{1}{H^2L^2}\right) - \left(H^2 - \frac{1}{L^2}\right) - \left(L^2 - \frac{1}{H^2}\right)$$

$$N_L = C_M\left(H^2L^2 - \frac{1}{H^2L^2}\right) - C_H\left(L^2 - \frac{1}{L^2}\right) - C_L\left(H^2 - \frac{1}{H^2}\right)$$

$$N_A = -C_M\left(L^2 - \frac{1}{H^2}\right) + C_H(L^2 - 1) + C_L\left(1 - \frac{1}{H^2}\right)$$

$$N_P = -C_M\left(H^2 - \frac{1}{L^2}\right) + C_H\left(1 - \frac{1}{L^2}\right) + C_L(H^2 - 1). \quad (94)$$

Then the values of Q_L, Q_A, and Q_P which apply at the frequency f_M are found from

$$Q_L = \Delta/N_L$$
$$Q_A = \Delta/N_A$$
$$Q_P = \Delta/N_P. \quad (95)$$

Using the same data, the total enclosure loss Q_B at the frequency f_M is

$$Q_B(f_M) = 1/C_M = Fk_M. \quad (96)$$

The approximate formula for $Q_B = Q_L$ given in Eq. (49) differs from Eq. (96) only in that R_{ES} is assumed constant, i.e., that $\rho_M = r_0$. However, because ρ_M is seldom very different from r_0, and particularly because $r_M - 1$ is usually much less than $\rho_M - 1$, Eq. (49) provides an adequately accurate measurement of total losses for normal evaluation purposes.

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